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# Student loans, fertility, and economic growth<sup>\*</sup>

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## Abstract

The cost of attaining higher education is growing in some developed countries. More young people borrow larger amounts than before to finance their higher education. Several media reports indicate that student loans might affect young people's decision making regarding important life events such as marriage, childbirth, purchasing a house, and so on. Specifically, this paper focuses on how the burden of student loans affects young people's decision making with regard to the number of children to have, and studies the fertility rate, gross domestic product (GDP) growth rate, and growth rate of GDP per capita using a three-period overlapping generations model. A young agent needs to borrow to accumulate his/her human capital, although for some reason, s/he faces the borrowing constraint. In the next period, the agent repays his/her debt as well as determines the number of children to have. Under this setting, this paper analyzes how the tightness of the borrowing constraints affects the growth rates of the population, GDP, and GDP per capita. The paper finds that when rearing children is time-consuming, the population growth rate decreases as the borrowing constraints are relaxed. Moreover, the paper shows a case in which the GDP growth rate decreases as the borrowing constraints are relaxed, whereas the growth rate of GDP per capita still increases.

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In addition, I show that if the cost of rearing children is mainly monetary, then the population growth rate is not necessarily decreasing as the borrowing constraints are relaxed. The paper also calibrates the model using U.S. data.

**Keywords:** Student loans, human capital accumulation, fertility, growth rate of GDP, growth rate of GDP per capita, overlapping generations model

**JEL Classification:** E44, I25, J13, J24

## 1 Introduction

Developed countries such as the U.S. and Japan report that the amount of student loans to attend universities and colleges has been increasing progressively.<sup>1</sup> Many people and the media believe that such a heavy burden could change young people's life styles by causing delays in marriage, leading them to have fewer children, and so on, and that such changes affect an economy (Read, for instance, Brown and Caldwell (2013), The New York Times (2013), The Los Angeles Times (2015), and The Wall Street Journal (2012)). Thus, this paper constructs a three-period overlapping generations (OLG) model to examine the effect of student loans on the population growth rate and economic growth.

The model in this paper is based on De Gregorio (1996) and Kitaura (2012). In the first period of life, a young agent is born, and s/he accumulates human capital. To finance the cost of human capital accumulation, the young agent borrows because s/he has no wealth. The agent, however, faces a borrowing constraint for some reason.<sup>2</sup> The amount that the agent can borrow has an upper bound, which is exogenously determined. In the second period, the middle-aged agent works, consumes, saves, repays his/her debt, and has children. In the third period, the old agent receives the return from savings, consumes it, and exits the economy. The agent derives utility from consumption and the number of children in his/her

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<sup>1</sup>For related data, see, for instance, Lochner and Moge-Naranjo (2014).

<sup>2</sup>This paper does not discuss how the upper bound is determined endogenously, although the issue is important. As for endogenous borrowing constraints, see, for instance, Andolfatto and Gervais (2006), de la Croix and Michel (2007), Wang (2014), and so on.

middle age and consumption in his/her old age.

The main question asked in this paper is how are the long-run economic growth rate and fertility rate affected if the borrowing constraint is relaxed? As noted above, a young agent needs to borrow to accumulate human capital. In this paper, I assume that s/he can borrow up to some ratio of labor income in his/her middle age. Relaxing the borrowing constraint implies that the young agent can borrow more, that is, the ratio becomes larger. Using a perfect foresight competitive equilibrium as the equilibrium concept, I prove that it is unique and characterize a unique balanced-growth path (BGP) equilibrium. In this unique BGP equilibrium, I investigate how the growth rates of aggregate output, output per capita, and population change as the borrowing constraint is relaxed.

First, I show that the fertility rate, or the population growth rate, in the BGP equilibrium strictly decreases as the borrowing constraints are relaxed until these constraints no longer bind. This paper assumes that the cost of raising a child is time. In addition, for tractability, the utility function is assumed to be additively separable of the natural log functions. Under this assumption, even if labor income increases owing to accumulating human capital, because the income effect and the substitution effect are cancelled out, economic growth does not affect the number of children a middle-aged agent has. As the borrowing constraints are relaxed, however, the repayment that the middle-aged agent has to make increases, and thus, the agent will have a smaller number of children as the borrowing constraints are relaxed. If the cost of rearing children is assumed to be monetary rather than time, relaxing borrowing constraints does not necessarily decrease the fertility rate. Since relaxing borrowing constraints increases human capital accumulation and stimulates economic growth, as long as a child is a normal good, the middle-aged agent will have more children. This suggests that lending young agents more will reduce the fertility rate if raising children is time-consuming rather than goods-consuming.

Second, I examine how the gross domestic product (GDP) growth rate in the BGP equilibrium changes if the borrowing constraints are relaxed. The size of the GDP growth rate in the BGP equilibrium is determined by the growth rates of human capital and the population. Since the population growth rate decreases as the borrowing constraints are relaxed, relaxing these constraints slows down the GDP growth rate. The growth rate of human capital depends on the tightness of the borrowing constraints

and capital per effective unit of labor through the labor wage and interest rate. Since a young agent can borrow more, s/he can accumulate more human capital. Thus, relaxing the borrowing constraints raises the GDP growth rate. On the other hand, if the young agent borrows more in the capital market, the supply of physical capital decreases, which increases the interest rate. At the same time, more human capital stock reduces the marginal product of the effective unit of labor, which leads to a decrease in labor wage. Since a young agent has to repay the debt in the next period from his/her labor income, higher interest rate and lower labor wage are disincentives against accumulating human capital for young agents. Thus, the growth rate of human capital in the BGP equilibrium will increase as the borrowing constraints are relaxed up to some point, and it will start decreasing as they are relaxed. Combining the effects of the population and human capital growth rates gives us a GDP growth rate that is hump-shaped in the degree of tightness of the borrowing constraints. Kitaura (2012) derived the same result in the setting where the population growth rate is exogenously given and constant.

Lastly, I investigate how the growth rate of GDP per capita will be affected if the borrowing constraints are relaxed. Since the growth rate of GDP per capita is GDP growth rate divided by the population growth rate, the growth rate of GDP per capita will increase up to some point as the borrowing constraints are relaxed, and will thereafter start decreasing with further relaxation of the constraints. In this paper's setting, I clarify the relationship between the GDP growth rate and growth rate of GDP per capita. I show that the peak of the GDP growth rate appears to the left of the peak of the growth rate of GDP per capita. This implies three possible cases, depending on parameter values.<sup>3</sup> In the first case, both growth rates are increasing as the borrowing constraints are relaxed. In the second case, both growth rates are increasing as the borrowing constraints are relaxed, and only the GDP growth rate starts decreasing after some point. In the third case, both growth rates increase as the borrowing constraints are relaxed, and the GDP growth rate starts decreasing, and later, the growth rate of GDP per capita also starts decreasing. The most interesting case might be the second one. Jappelli and Pagano (1994) were the first to examine the effect of the borrowing constraint to economic growth. Since they did not consider human

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<sup>3</sup>If the borrowing constraints are too relaxed, then both growth rates stay constant.

capital accumulation, borrowing more reduces the amount of capital formation, which affects economic growth negatively. De Gregorio (1996) pointed out the possibility that if an agent can borrow more and thus accumulate more human capital, then relaxing the borrowing constraint affects economic growth positively. Since De Gregorio (1996) considered a small open economy, the effects of the changes of the factor prices in factor markets were ignored, and it is possible that the results of De Gregorio (1996) might differ in a closed economy setting. Kitaura (2012) examined the same question as De Gregorio (1996) in a closed economy, and he showed that the GDP growth rate is not necessarily increasing as the borrowing constraints are relaxed, which is in disagreement with De Gregorio (1996). More precisely, he showed a case in which the GDP growth rate increases up to some point as the borrowing constraints are relaxed and starts decreasing after this point. In all of the above-mentioned papers, the population growth rate is exogenously given and constant. Thus, the growth rates of the GDP and GDP per capita are the same. The second case shows the possibility that even though GDP growth rate decreases as the borrowing constraints are relaxed, the growth rate of GDP per capita increases.

In addition to theoretical analysis, although the model is very simple, I calibrate the model under parameter values that describe the U.S. economy. The model suggests that relaxing the borrowing constraints can increase both the GDP growth rate and the growth rate of GDP per capita, and only the GDP growth rate starts decreasing after some point. Of course, the population growth rate decreases by relaxing the borrowing constraints.

The remainder of the paper is organized as follows. Section 2 describes the economic model, and Section 3 characterizes the BGP equilibrium. In Section 4, comparative statics is conducted, and Section 5 provides a numerical simulation. Section 6 discusses the case in which the cost of rearing children is monetary. Section 7 concludes the paper.

## 2 Model

Time is discrete and continues forever;  $t = 1, 2, \dots$ . An agent lives for three periods: young, middle, and old. Thus, in each period  $t$ , three generations coexist. If an agent is young in period, say,  $t - 1$ , s/he

decides the educational expenditure,  $e_{t-1}$ , to accumulate his/her human capital. Suppose that a young agent finances educational expenditure by borrowing. Thus, s/he will borrow  $e_{t-1}$  in period  $t - 1$  in the imperfect capital market. An agent who borrows  $e_{t-1}$  in period  $t - 1$  has to repay the debt in the next period with (gross) interest rate  $r_t$ . The imperfectness of the capital market is described by the borrowing constraint. Formally, a young agent faces the borrowing constraint,

$$e_{t-1}r_t \leq \mu w_t h_t, \quad (1)$$

where  $\mu \in (0, 1]$ . In some sense, this  $\mu$  reflects how frequently young agents default. If  $\mu$  is low, it means that because young agents default so often, middle agents cannot lend too much. If a young agent spends  $e_{t-1}$  for human capital accumulation in period  $t - 1$ , his/her human capital in period  $t$  will be

$$h_t = \theta e_{t-1}^\eta h_{t-1}^{1-\eta}, \quad (2)$$

where  $\theta > 0$  and  $\eta \in (0, 1)$ .

A middle agent in period  $t$  is endowed with one unit of time. S/he divides one unit of time between working and rearing children. To raise one child, a middle agent has to spend  $\phi > 0$  units of time. If a middle agent has  $n_t$  children, then the time cost for rearing them is  $\phi n_t$ . I assume that this time cost does not help a young agent accumulate his/her human capital at all. Letting  $w_t$  be a real wage, a middle agent with human capital  $h_t$  will receive the effective labor income,  $w_t h_t$ . Then, the budget constraint for a middle agent is

$$c_t + s_t + e_{t-1}r_t \leq (1 - \phi n_t)w_t h_t, \quad (3)$$

where  $c_t$  is the consumption, and  $s_t$  is the savings. When an agent becomes old, s/he retires and consumes  $d_{t+1}$  with interest income,  $r_{t+1}s_t$ . Thus, the budget constraint for an old agent is

$$d_{t+1} \leq r_{t+1}s_t. \quad (4)$$

Assume that  $s_0 > 0$ ,  $e_0 > 0$ , and  $h_0 > 0$  are given exogenously. From Equations (3) and (4), the lifetime budget constraint is

$$c_t + \frac{d_{t+1}}{r_{t+1}} + e_{t-1}r_t \leq (1 - \phi n_t)w_t h_t. \quad (5)$$

Let  $N_t$  be the population of middle agents in period  $t$ . If the middle agents in period  $t$  have  $n_t$  children, then the population of the middle agents in period  $t + 1$  is

$$N_{t+1} = n_t N_t.$$

Assume that  $N_0 > 0$  and  $n_0 > 0$  are given exogenously.

An agent derives utility from consumption and the number of children when s/he is middle-aged, and from consumption when s/he is old. Hence, an agent's lifetime utility is expressed by

$$U(c_t, d_{t+1}, n_t) := \ln(c_t) + \gamma \ln(n_t) + \beta \ln(d_{t+1}),$$

where  $\beta > 0$  and  $\gamma > 0$  stand for the preference strength relative to the consumption when the agent is middle-aged.

There is a representative firm in the economy. Its production function is expressed as

$$Y_t = AF(K_t, L_t) := AK_t^\alpha L_t^{1-\alpha},$$

where  $\alpha \in (0, 1)$ , and  $K_t$  and  $L_t$  are the aggregate capital stock and aggregate effective labor in the economy, respectively. Given the real wage,  $w_t$ , and the real rental rate of capital,  $r_t$ , the firm's profit in period  $t$  is

$$AK_t^\alpha L_t^{1-\alpha} - r_t K_t - w_t L_t.$$

I assume that capital is fully depreciated after production. Let  $k_t := K_t/L_t$  denote capital per effective unit of labor in period  $t$ .

## 2.1 Equilibrium

The equilibrium concept is a standard *perfect foresight competitive equilibrium*. The formal definition is as follows.

**Definition 2.1.** Given  $N_0 > 0$ ,  $n_0 > 0$ ,  $s_0 > 0$ ,  $e_0 > 0$ , and  $h_0 > 0$ , an **equilibrium** consists of a consumption sequence,  $(d_1, (c_t, d_{t+1})_{t=1}^\infty)$ ; a sequence of fertility,  $(n_t)_{t=1}^\infty$ ; a sequence of educational expenditure,



$(e_t)_{t=1}^\infty$ ; a sequence of savings,  $(s_t)_{t=1}^\infty$ ; a sequence of inputs for production,  $(K_t, L_t)_{t=1}^\infty$ ; and a sequence of prices  $(w_t, r_t)_{t=1}^\infty$  such that

1. given the prices, a middle-aged agent solves

$$\begin{aligned} \max_{c_t, e_{t-1}, n_t, d_{t+1}} \quad & \ln(c_t) + \gamma \ln(n_t) + \beta \ln(d_{t+1}) \\ \text{s.t.} \quad & c_t + \frac{d_{t+1}}{r_{t+1}} + e_{t-1}r_t \leq (1 - \phi n_t)w_t h_t, \\ & h_t = \theta e_{t-1}^\eta h_{t-1}^{1-\eta}, \\ & e_{t-1}r_t \leq \mu w_t h_t. \end{aligned}$$

2. Given prices  $(w_1, r_2)$ ,  $e_0 > 0$ , and  $h_0 > 0$ , the initial middle-aged agent solves

$$\begin{aligned} \max_{c_1, n_1, d_2} \quad & \ln(c_1) + \gamma \ln(n_1) + \beta \ln(d_2) \\ \text{s.t.} \quad & c_1 + \frac{d_2}{r_2} + e_0 r_1 \leq (1 - \phi n_1)w_1 h_1, \\ & h_1 = \theta e_0^\eta h_0^{1-\eta}. \end{aligned}$$

3. For the initial old agent,  $d_1 = r_1 s_0$ .

4. The firm maximizes its profit in each period such that

$$w_t = (1 - \alpha)A k_t^\alpha, \quad r_t = \alpha A k_t^{\alpha-1}.$$

5. A capital market and a labor market clear in each period: For all  $t$ ,  $K_t = N_{t-1}s_{t-1} - N_t e_{t-1}$  and  $L_t = N_t(1 - \phi n_t)h_t$ , where  $N_t = n_{t-1}N_{t-1}$  and  $h_t = \theta e_{t-1}^\eta h_{t-1}^{1-\eta}$  are satisfied.

A BGP equilibrium is an equilibrium such that for some  $g > 0$ ,  $\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{L_{t+1}}{L_t} = g$  for all  $t$ .

### 3 Characterizing equilibrium

The first-order conditions to a young agent's problem induce

$$d_{t+1} = \beta r_{t+1} c_t, \tag{6}$$

$$\phi n_t w_t h_t = \gamma c_t. \tag{7}$$

Letting  $\lambda_1$  and  $\lambda_2$  be the Lagrange multipliers for (5) and (1), respectively, another first-order condition is

$$-\lambda_1 \left[ r_t - (1 - \phi n_t) w_t \theta \eta e_{t-1}^{\eta-1} h_{t-1}^{1-\eta} \right] = \lambda_2 \left[ r_t - \mu w_t \theta \eta e_{t-1}^{\eta-1} h_{t-1}^{1-\eta} \right]. \quad (8)$$

### 3.1 Borrowing constraints do not bind

Consider an equilibrium where the borrowing constraints do not bind. That is,  $\lambda_2 = 0$ . Since  $\lambda_1 > 0$ , from Equation (8),

$$r_t = (1 - \phi n_t) w_t \theta \eta e_{t-1}^{\eta-1} h_{t-1}^{1-\eta} \quad (9)$$

holds. From this equation,

$$r_t e_{t-1} = \eta (1 - \phi n_t) w_t h_t \quad (10)$$

holds. Using Equations (5), (6), (7), and (10),

$$c_t = \frac{1 - \eta}{1 + \beta + \gamma(1 - \eta)} w_t h_t$$

holds. From this,

$$n_t = \bar{n}^{nb} := \frac{\gamma(1 - \eta)}{\phi[1 + \beta + \gamma(1 - \eta)]}$$

holds, where the superscript “nb” stands for “not bind.” Note that the fertility rate is constant over time.

Plugging  $n_t$  into Equation (9),

$$e_{t-1} = \left[ \frac{\theta \eta (1 + \beta)}{1 + \beta + \gamma(1 - \eta)} \frac{w_t}{r_t} \right]^{\frac{1}{1-\eta}} h_{t-1} \quad (11)$$

holds.

From the law of motion of human capital, in this case,

$$\frac{h_t}{h_{t-1}} = \theta \left[ \frac{\theta \eta (1 + \beta)}{1 + \beta + \gamma(1 - \eta)} \frac{1 - \alpha}{\alpha} k_t \right]^{\frac{\eta}{1-\eta}} \quad (12)$$

is satisfied, where I use equilibrium conditions  $w_t = A(1 - \alpha)k_t^\alpha$  and  $r_t = A\alpha k_t^{\alpha-1}$ .

From the market clearing condition,

$$k_{t+1} = \frac{K_{t+1}}{L_{t+1}} = \frac{N_t s_t - N_{t+1} e_t}{N_{t+1}(1 - \phi n_{t+1})h_{t+1}} = \frac{s_t}{n_t(1 - \phi n_{t+1})h_{t+1}} - \frac{e_t}{(1 - \phi n_{t+1})h_{t+1}}.$$

The amount of savings is

$$s_t = \frac{d_{t+1}}{r_{t+1}} = \beta c_t = \frac{\beta(1 - \eta)}{1 + \beta + \gamma(1 - \eta)} w_t h_t.$$

Thus,

$$k_{t+1} = \frac{\beta(1 - \eta)A(1 - \alpha)k_t^\alpha}{1 + \beta + \gamma(1 - \eta)} \frac{1}{\bar{n}^{nb}(1 - \phi \bar{n}^{nb})} \frac{h_t}{h_{t+1}} - \frac{1}{1 - \phi \bar{n}^{nb}} \left[ \frac{\theta \eta(1 + \beta)}{1 + \beta + \gamma(1 - \eta)} \frac{1 - \alpha}{\alpha} k_{t+1} \right]^{\frac{1}{1 - \eta}} \frac{h_t}{h_{t+1}}.$$

Applying Equation (12) to this equation,

$$k_{t+1} \theta \left[ \frac{\theta \eta(1 + \beta)}{1 + \beta + \gamma(1 - \eta)} \frac{1 - \alpha}{\alpha} k_{t+1} \right]^{\frac{\eta}{1 - \eta}} = \frac{\beta(1 - \eta)A(1 - \alpha)k_t^\alpha}{1 + \beta + \gamma(1 - \eta)} \frac{1}{\bar{n}^{nb}(1 - \phi \bar{n}^{nb})} - \frac{1}{1 - \phi \bar{n}^{nb}} \left[ \frac{\theta \eta(1 + \beta)}{1 + \beta + \gamma(1 - \eta)} \frac{1 - \alpha}{\alpha} k_{t+1} \right]^{\frac{1}{1 - \eta}}.$$

After rearranging this equation, I obtain

$$k_{t+1} = \frac{1}{\theta \left( \frac{\eta(1 + \beta)}{1 + \beta + \gamma(1 - \eta)} \frac{1 - \alpha}{\alpha} \right)^\eta \left[ \frac{\alpha + \eta(1 - \alpha)}{\alpha} \right]^{1 - \eta}} \left[ \frac{\beta(1 - \eta)A(1 - \alpha)}{1 + \beta + \gamma(1 - \eta)} \frac{1}{\bar{n}^{nb}(1 - \phi \bar{n}^{nb})} \right]^{1 - \eta} k_t^{\alpha(1 - \eta)}.$$

This gives us a unique, globally stable steady state,  $k$ , which is characterized by

$$\bar{k}^{nb} := \left[ \frac{1}{\theta \left( \frac{\eta(1 + \beta)}{1 + \beta + \gamma(1 - \eta)} \frac{1 - \alpha}{\alpha} \right)^\eta \left[ \frac{\alpha + \eta(1 - \alpha)}{\alpha} \right]^{1 - \eta}} \left[ \frac{\beta(1 - \eta)A(1 - \alpha)}{1 + \beta + \gamma(1 - \eta)} \frac{1}{\bar{n}^{nb}(1 - \phi \bar{n}^{nb})} \right]^{1 - \eta} \right]^{\frac{1}{1 - \alpha(1 - \eta)}}.$$

From Equation (12), the growth rate of human capital in the long-run equilibrium is

$$\hat{g}^{nb} := \frac{h_{t+1}}{h_t} = Z^{nb} \left( \frac{\eta}{1 + \beta + \gamma(1 - \eta)} \right)^{\frac{\eta(1 - \alpha)}{1 - \alpha(1 - \eta)}} \left[ \frac{1 + \beta + \gamma(1 - \eta)}{\alpha + \eta(1 - \alpha)} \right]^{\frac{\eta}{1 - \alpha(1 - \eta)}},$$

where  $Z^{nb} := \theta^{\frac{1-\alpha}{1-\alpha(1-\eta)}} \left( \frac{1-\alpha}{\alpha} \right)^{\frac{\eta(1-\alpha)}{1-\alpha(1-\eta)}} \left[ \frac{\beta A(1-\alpha)\alpha\phi}{\gamma(1+\beta)^\alpha} \right]^{\frac{\eta}{1-\alpha(1-\eta)}}$ . Note that the growth rate of aggregate output is  $\frac{Y_{t+1}}{Y_t} = \left( \frac{K_{t+1}}{K_t} \right)^\alpha \left( \frac{L_{t+1}}{L_t} \right)^{1-\alpha} = \left( \frac{k_{t+1}}{k_t} \right)^\alpha \left( \frac{(1-\phi)n_{t+1}N_{t+1}h_{t+1}}{(1-\phi)n_tN_th_t} \right)$ . Hence, the growth rate of aggregate output in the BGP equilibrium,  $g^{nb}$ , is

$$g^{nb} = \bar{n}^{nb} \hat{g}^{nb} = \hat{Z}^{nb} \frac{1-\eta}{1+\beta+\gamma(1-\eta)} \left( \frac{\eta}{1+\beta+\gamma(1-\eta)} \right)^{\frac{\eta(1-\alpha)}{1-\alpha(1-\eta)}} \left[ \frac{1+\beta+\gamma(1-\eta)}{\alpha+\eta(1-\alpha)} \right]^{\frac{\eta}{1-\alpha(1-\eta)}},$$

where  $\hat{Z}^{nb} := Z^{nb} \frac{\gamma}{\phi}$ . Let  $y_t$  be output per capita, that is,  $y_t := \frac{Y_t}{N_{t-1}+N_t+N_{t+1}}$ . In the BGP equilibrium, the growth rate of output per capita is

$$\frac{y_{t+1}}{y_t} = \frac{Y_{t+1}}{Y_t} \frac{N_{t-1}+N_t+N_{t+1}}{N_t+N_{t+1}+N_{t+2}} = \frac{g^{nb}}{\bar{n}^{nb}} = \hat{g}^{nb}.$$

### 3.2 Borrowing constraints bind

When borrowing constraints bind,  $\lambda_2 > 0$ . From the borrowing constraint,

$$e_{t-1} = \mu \frac{w_t}{r_t} h_t = \mu \frac{1-\alpha}{\alpha} k_t h_t = \mu \frac{1-\alpha}{\alpha} k_t \theta e_{t-1}^\eta h_{t-1}^{1-\eta}.$$

Thus,

$$e_{t-1} = \left[ \mu \theta \frac{1-\alpha}{\alpha} k_t \right]^{\frac{1}{1-\eta}} h_{t-1}. \quad (13)$$

From Equations (5), (6), and (7),

$$c_t = \frac{(1-\mu)w_t h_t}{1+\beta+\gamma}.$$

From Equation (7), the fertility rate is

$$n_t = \bar{n}^b := \frac{\gamma(1-\mu)}{\phi(1+\beta+\gamma)},$$

where the superscript “ $b$ ” stands for “binds.” From the law of motion of human capital and Equation (13),

$$\frac{h_t}{h_{t-1}} = \theta \left[ \frac{\mu \theta (1-\alpha)}{\alpha} k_t \right]^{\frac{\eta}{1-\eta}}.$$

When do borrowing constraints bind? The borrowing constraints bind if and only if  $e_{t-1}$ , defined by Equation (13), is smaller than  $e_{t-1}$ , defined by Equation (11), that is,

$$\left[ \mu \theta \frac{1-\alpha}{\alpha} k_t \right]^{\frac{1}{1-\eta}} h_{t-1} < \left[ \frac{\theta \eta (1+\beta)}{1+\beta+\gamma(1-\eta)} \frac{1-\alpha}{\alpha} k_t \right]^{\frac{1}{1-\eta}} h_{t-1}$$

holds. Thus, the borrowing constraints bind if and only if

$$\mu < \bar{\mu} := \frac{\eta(1+\beta)}{1+\beta+\gamma(1-\eta)}.$$

**Lemma 3.1.** (i)  $\bar{\mu}$  is strictly increasing in  $\beta$  and strictly decreasing in  $\gamma$ . (ii)  $\bar{\mu}$  is strictly increasing in  $\eta$ , and  $\bar{\mu}$  is 0 when  $\eta = 0$  and 1 when  $\eta = 1$ .

*Proof.* See the Appendix. Q.E.D.

Intuitively, if the utility from children is relatively more important compared to the utility from consumption in the middle and old periods, an agent will want to have more children when s/he is middle-aged. Since the agent has to spend time on rearing children, time for work is reduced. Thus, the return from investing human capital becomes lower, and the agent invests less if  $\gamma$  is larger. This implies that even if the borrowing constraint is relaxed, it may not affect an agent's human capital investment at all when the utility weight on having children is relatively large compared to that on consumption in the middle and old periods. When  $\eta$  is large, the return from human capital investment is large. Thus, an agent has an incentive to invest in his/her human capital more if  $\eta$  is large. This implies that the borrowing constraint binds frequently.

The amount of savings is

$$s_t = \frac{d_{t+1}}{r_{t+1}} = \beta c_t = \frac{\beta(1-\mu)}{1+\beta+\gamma} w_t h_t.$$

Since  $k_{t+1} = \frac{s_t}{n_t(1-\phi_{t+1})h_{t+1}} - \frac{e_t}{(1-\phi_{t+1})h_{t+1}}$ , using the algebra analogous to the previous “not binding” case gives us

$$k_{t+1} = \left[ \frac{\frac{\beta A(1-\alpha)\phi}{\gamma}}{\left[ \frac{\mu \theta (1-\alpha)}{\alpha} \right]^{\frac{\eta}{1-\eta}} \theta \frac{(1+\beta)[\alpha+\mu(1-\alpha)]+\gamma\mu}{\alpha(1+\beta+\gamma)}} \right]^{1-\eta} k_t^{\alpha(1-\eta)}.$$

From this, the steady-state capital per effective unit of labor is

$$\bar{k}^b := \left[ \frac{\frac{\beta A(1-\alpha)\phi}{\gamma}}{\left[ \frac{\mu\theta(1-\alpha)}{\alpha} \right]^{\frac{\eta}{1-\eta}} \theta \frac{(1+\beta)[\alpha+\mu(1-\alpha)]+\gamma\mu}{\alpha(1+\beta+\gamma)}} \right]^{\frac{1-\eta}{1-\alpha(1-\eta)}}.$$

Thus, the growth rate of human capital in the BGP equilibrium is

$$\hat{g}^b := \frac{h_{t+1}}{h_t} = \theta \left[ \frac{\mu\theta(1-\alpha)}{\alpha} \bar{k}^b \right]^{\frac{\eta}{1-\eta}} = Z^b \left[ \frac{\mu^{1-\alpha}}{(1+\beta)\alpha + \mu[(1+\beta)(1-\alpha) + \gamma]} \right]^{\frac{\eta}{1-\alpha(1-\eta)}},$$

where  $Z^b := \theta^{\frac{1-\alpha}{1-\alpha(1-\eta)}} \left( \frac{1-\alpha}{\alpha} \right)^{\frac{\eta(1-\alpha)}{1-\alpha(1-\eta)}} \left( \frac{\beta A(1-\alpha)\phi\alpha(1+\beta+\gamma)}{\gamma} \right)^{\frac{\eta}{1-\alpha(1-\eta)}}$ . Notice that this growth rate is equivalent to the growth rate of output per capita.

From this, the growth rate in the BGP equilibrium is

$$g^b = \bar{n}^b \hat{g}^b = \hat{Z}^b (1-\mu) \left[ \frac{\mu^{1-\alpha}}{(1+\beta)\alpha + \mu[(1-\alpha)(1+\beta) + \gamma]} \right]^{\frac{\eta}{1-\alpha(1-\eta)}},$$

where  $\hat{Z}^b := Z^b \frac{\gamma}{\phi(1+\beta+\gamma)}$ .

## 4 Results

The first result concerns the fertility rate.

**Proposition 4.1.** *The population growth rate, or the fertility rate, in the BGP equilibrium is decreasing in  $\mu$  when  $\mu < \hat{\mu}$  and is constant in  $\mu$  when  $\mu \geq \hat{\mu}$ .*

*Proof.* See the Appendix. Q.E.D.

As shown in this result, the fertility rate decreases as the borrowing constraint relaxes. I later show that the economy will grow as  $\mu$  increases. Although the economy grows, the fertility rate shrinks. Furthermore, the main reason for this reduction of the fertility rate is the burden of student loan. Because the income effect and substitution effect are cancelled out in this example economy, an increase in wage

due to economic growth does not affect the fertility rate at all. Thus, this decrease in the fertility rate is purely due to the burden of the student loan.

The second result concerns the aggregate output growth rate. Recall that when  $\mu \geq \bar{\mu}$ , the output growth rate is constant at  $g^{nb}$ .

**Proposition 4.2.** *There exists a unique  $\tilde{\mu} \in (0, 1)$  such that*

$$\frac{dg^b}{d\mu} \begin{cases} > \\ = \\ < \end{cases} 0 \text{ if } \mu \begin{cases} < \\ = \\ > \end{cases} \tilde{\mu}.$$

*Proof.* See the Appendix.

*Q.E.D.*

When  $\tilde{\mu} \leq \bar{\mu}$ , the growth rate of  $Y$  in the BGP equilibrium is increasing in  $\mu$  up to  $\tilde{\mu}$ , decreasing in  $\mu$  for  $\mu \in (\tilde{\mu}, \bar{\mu})$ , and constant in  $\mu$  for  $\mu \geq \bar{\mu}$ . When  $\tilde{\mu} > \bar{\mu}$ , the growth rate of  $Y$  is increasing in  $\mu$  up to  $\bar{\mu}$ , and constant in  $\mu$  for  $\mu \geq \bar{\mu}$ . An increase in  $\mu$  raises human capital accumulation, which has a positive effect on economic growth. At the same time, however, an increase in  $\mu$  decreases the population and amount of physical capital stock, which has a negative effect on economic growth. When  $\mu$  is small, the first positive effect dominates the second negative effect, whereas as  $\mu$  becomes large, the first effect disappears and the economic growth rate becomes smaller. This finding is in agreement with the result of Kitaura (2012), who assumed the population growth rate is constant.

With Lemma 3.1, I show that there exists a unique  $\tilde{\eta} \in (0, 1)$  such that  $\bar{\mu}(\tilde{\eta}) = \tilde{\mu}$ . In summary,

**Remark 4.1.** *there exists a unique  $\tilde{\eta} \in (0, 1)$  such that (i) if  $\eta \leq \tilde{\eta}$ , then the growth rate of  $Y$  is increasing in  $\mu$  up to  $\bar{\mu}$  and stays constant in  $\mu$  for  $\mu \geq \bar{\mu}$ , (ii) if  $\eta > \tilde{\eta}$ , then the growth rate of  $Y$  is increasing in  $\mu$  up to  $\tilde{\mu}$ , decreasing in  $\mu$  for  $\mu \in (\tilde{\mu}, \bar{\mu})$ , and stays constant in  $\mu$  for  $\mu \geq \bar{\mu}$ .*

The third result relates to the growth rate of output per capita. Let  $y$  be the output per capita. Note again that when  $\mu \geq \bar{\mu}$ , the growth rate of  $y$  is constant in  $\mu$ .

**Proposition 4.3.** Letting  $\hat{\mu} := \frac{(1-\alpha)(1+\beta)}{(1+\beta)(1-\alpha)+\gamma}$ ,

$$\frac{d\hat{g}^b}{d\mu} \begin{cases} > \\ = \\ < \end{cases} 0 \text{ if } \mu \begin{cases} < \\ = \\ > \end{cases} \hat{\mu}.$$

*Proof.* See the Appendix. Q.E.D.

When  $\hat{\mu} \leq \bar{\mu}$ , the output growth rate of  $y$  in the BGP equilibrium is increasing in  $\mu$  up to  $\hat{\mu}$ , decreasing in  $\mu$  for  $\mu \in (\hat{\mu}, \bar{\mu})$ , and constant in  $\mu$  for  $\mu > \bar{\mu}$ . When  $\hat{\mu} > \bar{\mu}$ , the output growth rate of  $y$  is increasing in  $\mu$  up to  $\bar{\mu}$ , and stays constant in  $\mu$  for  $\mu > \bar{\mu}$ , which is quite similar to the growth rate of  $Y$ . However, the cutoff levels of  $\mu$  ( $\tilde{\mu}$  and  $\hat{\mu}$ ) are different for the two growth rates. It can be shown that  $\tilde{\mu} < \hat{\mu}$ .

**Lemma 4.1.**  $0 < \tilde{\mu} < \hat{\mu} < 1$ .

*Proof.* See the Appendix. Q.E.D.

By Lemma 3.1, there exists a unique  $\hat{\eta} \in (0, 1)$  such that  $\bar{\mu}(\hat{\eta}) = \hat{\mu}$ . Since  $\bar{\mu}(\eta)$  is strictly increasing, Lemma 4.1 implies that  $\tilde{\eta} < \hat{\eta}$ .

**Proposition 4.4.** (i) If  $\eta \leq \tilde{\eta}$ , then the growth rates of both  $Y$  and  $y$  are strictly increasing in  $\mu$  up to  $\bar{\mu}$ , and stay constant in  $\mu$  for  $\mu > \bar{\mu}$ . (ii) If  $\eta \in (\tilde{\eta}, \hat{\eta}]$ , then the growth rates of both  $Y$  and  $y$  are strictly increasing in  $\mu$  up to  $\tilde{\mu}$ , while the growth rate of  $Y$  is strictly decreasing and that of  $y$  is strictly increasing in  $\mu$  for  $\mu \in (\tilde{\mu}, \bar{\mu})$ . Further, both growth rates stay constant in  $\mu$  for  $\mu > \bar{\mu}$ . (iii) If  $\eta > \hat{\eta}$ , then the growth rate of both  $Y$  and  $y$  are strictly increasing in  $\mu$  up to  $\tilde{\mu}$ , the growth rate of  $Y$  is strictly decreasing and that of  $y$  is strictly increasing in  $\mu$  for  $\mu \in (\tilde{\mu}, \bar{\mu})$ , and both growth rates stay constant in  $\mu$  for  $\mu > \bar{\mu}$ .

When  $\eta$  is small, the effects of the borrowing constraints on the growth rates of  $Y$  and on  $y$  are the same, that is, they are strictly increasing in  $\mu$  up to  $\bar{\mu}$  and stay constant after that. When  $\eta$  is high, both growth rates are hump-shaped in  $\mu$ , while the  $\mu$ s for the highest growth rates are different. The most



interesting case might be that of  $\eta$  taking a medium value. When this happens, the growth rate of  $Y$  is hump-shaped in  $\mu$ , whereas the growth rate of  $y$  is strictly increasing in  $\mu$ . This implies that although the aggregate output grows slowly when the borrowing constraint relaxes, the output per capita grows quickly. This possibility is ignored in the literature (e.g., De Gregorio (1996) and Kitaura (2012)). De Gregorio (1996) argued that an increase in  $\mu$  raises the growth rate of  $Y$ , and Kitaura (2012) argued that if factor prices are determined by the markets, the argument in De Gregorio (1996) is not necessarily true. In the literature, the (net) population growth rate is 0. Thus, the growth rates of aggregate output and output per capita are the same in past studies. This paper, however, shows that if the population growth rate is endogenously determined, the manner in which the borrowing constraint affects the growth rates of  $Y$  and  $y$  is not necessarily consistent.

## 5 Numerical simulation

This section calibrates the model using appropriate parameter values. Following Žamac (2007), I set  $\beta$  at  $(0.98)^{27}$ ,  $\alpha = \frac{1}{3}$ ,  $A = 21.6$ ,  $\theta = 1$ , and  $\eta = 0.16$ . Following de la Croix and Doepke (2003), I assume that the weight on fertility relative to consumption for the young ( $\gamma$ ) is 0.271. The time cost for rearing a child,  $\phi$ , is set at 0.089. According to the American Time Use Survey, in 2014,<sup>4</sup> when both spouses worked full time, on average, mothers and fathers spent 5.21 hours and 6.06 hours for “working and work-related activities,” respectively, and mothers and fathers spent 1.35 hours and 0.88 hours over “caring for and helping household children,” respectively, in a day. In addition, according to the United States Census Bureau, in 2014, the average number of own children under 18 in families was 1.87.<sup>5</sup> Thus, we can consider that each household spends at least  $\frac{1.35+0.90}{1.87} = 1.2085$  hours to raise one child. In this numerical simulation, an agent in the model is regarded as a household. Since a household splits 1 unit of time between work and child-rearing,  $\phi$  should be set at  $\frac{1.2085}{11.27+2.26} \approx 0.089$ .

Under these parameter values,  $\bar{\mu} = 0.1311$  and  $\hat{\mu} = 0.7974$ . In addition, the growth rate of  $Y$  is

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<sup>4</sup>See [http://www.bls.gov/tus/tables/a7\\_0913.htm](http://www.bls.gov/tus/tables/a7_0913.htm).

<sup>5</sup>See <http://www.census.gov/hhes/families/data/cps2014AVG.html>.

maximized at  $\tilde{\mu} = 0.0913$ . The growth rate of  $Y$  is plotted as a solid line and that of  $y$  is plotted as a dashed line with dots in Figure 1. As Proposition 4.4 shows, the growth rate of  $Y$  is hump-shaped in  $\mu$ , whereas that of  $y$  is increasing in  $\mu$ , both of which are observed in Figure 1. Figure 2 shows how the fertility rate changes in  $\mu$ . Using U.S. population data provided by the World Bank, the average annual population growth rate between 1960 and 2014 is 1.016. Hence, the population growth rate over 27 years is around 1.5351. In this simulation, the fertility rate or population growth rate is 1.5351 when  $\mu$  is about 0.065. Since  $0.065 < 0.0913 = \tilde{\mu} < 0.1311 = \bar{\mu} < 0.7974 = \hat{\mu}$ , relaxing the borrowing constraint can simultaneously increase the growth rates of  $Y$  and  $y$ , although the population growth rate shrinks.

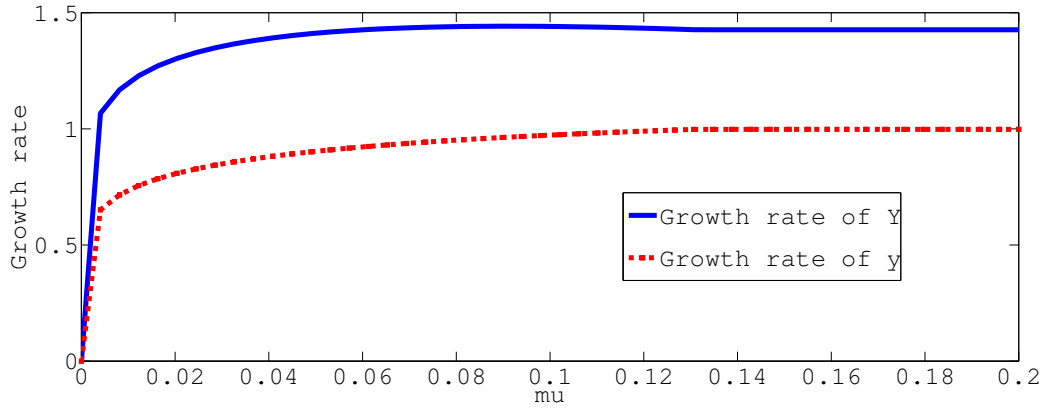


Figure 1: Growth rates of  $Y$  and  $y$  for different  $\mu$

## 6 Monetary cost of child-rearing

Raising a child needs not only a parent's time but also money. Several researchers such as Boldrin and Jones (2002), Fanti and Gori (2012), and Miyazaki (2013) considered both time cost and monetary cost to analyze the model. So far, this paper only focuses on the time cost of child-rearing, and this setting is necessary to derive Proposition 4.1, that is, the fertility rate is decreasing as the borrowing constraints are relaxed. In this section, I briefly discuss how the results will change if I use the monetary cost of

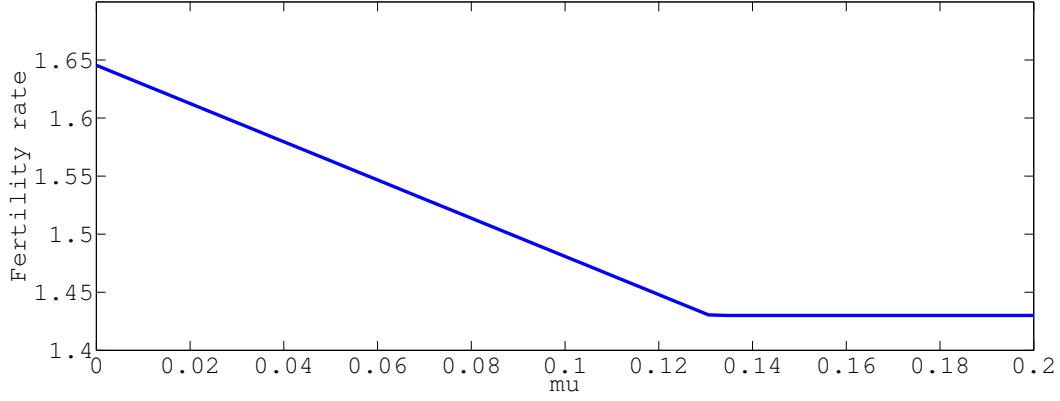


Figure 2: Fertility rate for different  $\mu$

rearing a child instead of the time cost.

Letting  $\psi$  be the monetary cost of raising one child, the lifetime budget constraint, Equation (5), is rewritten as

$$c_t + \frac{d_{t+1}}{r_{t+1}} + \psi n_t + r_t e_{t-1} \leq w_t h_t. \quad (14)$$

The other parts do not change. The first-order conditions to an agent's problem are

$$d_{t+1} = \beta r_{t+1} c_t, \quad (15)$$

$$\psi n_t = \gamma c_t, \quad (16)$$

$$-\lambda_1 \left[ r_t - w_t \theta \eta e_{t-1}^{\eta-1} h_{t-1}^{1-\eta} \right] = \lambda_2 \left[ r_t - \mu w_t \theta \eta e_{t-1}^{\eta-1} h_{t-1}^{1-\eta} \right]. \quad (17)$$

## 6.1 Borrowing constraints do not bind

In this case,  $\lambda_2 = 0$ . Thus, from Equation (17),

$$e_{t-1} = \left( \frac{\theta \eta w_t}{r_t} \right)^{\frac{1}{1-\eta}} h_{t-1}. \quad (18)$$

Analogous to Section 3, I have

$$c_t = \frac{1}{1+\beta+\gamma} \left( \frac{\theta\eta w_t}{r_t} \right)^{\frac{1}{1-\eta}} \left( \frac{1}{\eta} - 1 \right) h_{t-1} r_t, \quad (19)$$

$$n_t = \frac{\gamma}{\psi(1+\beta+\gamma)} \left( \frac{\theta\eta w_t}{r_t} \right)^{\frac{1}{1-\eta}} \left( \frac{1}{\eta} - 1 \right) h_{t-1} r_t, \quad (20)$$

$$s_t = \frac{\beta}{1+\beta+\gamma} \left( \frac{\theta\eta w_t}{r_t} \right)^{\frac{1}{1-\eta}} \left( \frac{1}{\eta} - 1 \right) h_{t-1} r_t. \quad (21)$$

From the capital market clearing condition, I get

$$k_{t+1} = \frac{s_t}{n_t h_{t+1}} - \frac{e_t}{h_{t+1}} = \frac{\beta\psi}{\gamma\theta e_t^\eta h_t^{1-\eta}} - \frac{(1-\alpha)\eta}{\alpha} k_{t+1},$$

where I replace  $w_t$  and  $r_t$  by  $A(1-\alpha)k_t^\alpha$  and  $A\alpha k_t^{\alpha-1}$ , respectively. From this,

$$k_{t+1} = \frac{\beta\psi}{\gamma \left[ 1 + \frac{1-\alpha}{\alpha} \eta \right] h_{t+1}}. \quad (22)$$

From the formulation of human capital and Equation (2), in equilibrium,

$$h_{t+1} = \theta \left( \frac{\theta\eta w_{t+1}}{r_{t+1}} \right)^{\frac{\eta}{1-\eta}} h_t = \theta \left( \frac{\theta\eta(1-\alpha)}{\alpha} \right)^{\frac{\eta}{1-\eta}} k_{t+1}^{\frac{\eta}{1-\eta}} h_t.$$

Plugging Equation (22) into this equation, I get

$$h_{t+1} = \theta^{1-\eta} \left( \frac{\theta\eta(1-\alpha)}{\alpha} \right)^\eta \left( \frac{\beta\psi}{\gamma \left[ 1 + \frac{(1-\alpha)\eta}{\alpha} \right]} \right)^\eta h_t^{1-\eta}. \quad (23)$$

From this equation and Equation (22), there is a unique BGP equilibrium, and the GDP growth rate in the BGP equilibrium is

$$\frac{Y_{t+1}}{Y_t} = \left( \frac{K_{t+1}}{K_t} \right)^\alpha \left( \frac{N_{t+1}h_{t+1}}{N_t h_t} \right)^{1-\alpha} = \left( \frac{k_{t+1}}{k_t} \right)^\alpha \frac{N_{t+1}h_{t+1}}{N_t h_t} = n_t := \bar{n}, \quad (24)$$

where  $\bar{n}$  is derived from Equations (16), (22), and (23). Thus, the growth rate of GDP per capita in the BGP equilibrium is 1.

## 6.2 Borrowing constraints bind

Following the same process used in the previous section,

$$k_{t+1} = \frac{\beta \psi}{\gamma \left[ 1 + \frac{\theta \mu (1-\alpha)}{\alpha} \right] h_{t+1}}, \quad (25)$$

$$h_{t+1} = \theta^{1-\eta} \left( \frac{\theta \mu (1-\alpha)}{\alpha} \right)^\eta \left( \frac{\beta \psi \alpha}{\gamma [\alpha + \mu \theta (1-\alpha)]} \right)^\eta h_t^{1-\eta} \quad (26)$$

are derived. Again, there is a unique BGP equilibrium. In the BGP equilibrium, the fertility rate is

$$\begin{aligned} \bar{n}^b &:= M \left( \frac{1}{\alpha + \mu \theta (1-\alpha)} \right)^{\frac{\eta + \alpha(1-\eta)}{1-\eta}} (1-\mu) \mu^{\frac{\eta}{1-\eta}} \left[ \frac{\mu}{\alpha + \mu \theta (1-\alpha)} \right]^{1-\alpha-\frac{\eta}{1-\eta}} \\ &= M \frac{(1-\mu) \mu^{1-\alpha}}{\alpha + \mu \theta (1-\alpha)}, \end{aligned} \quad (27)$$

where  $M := \frac{\gamma \theta (1-\alpha)}{\psi(1+\beta+\gamma)} \left( \frac{\theta(1-\alpha)}{\alpha} \right)^{\frac{\eta}{1-\eta}} \left( \frac{\alpha \beta \psi}{\gamma} \right)^{\frac{\eta + \alpha(1-\eta)}{1-\eta}} \left[ \theta^{\frac{1}{\eta}} \frac{\beta \psi (1-\alpha)}{\gamma} \right]^{1-\alpha-\frac{\eta}{1-\eta}}$ . The borrowing constraints bind if and only if  $\mu \leq \eta$ , which is the same as in Kitaura (2012). Note again that the GDP growth rate in the BGP equilibrium is  $\bar{n}^b$ .

**Proposition 6.1.** *There exists a unique  $\hat{\mu} \in (0, 1)$  such that*

$$\frac{d\bar{n}^b}{d\mu} \begin{cases} > \\ = \\ < \end{cases} 0 \text{ if } \mu \begin{cases} < \\ = \\ > \end{cases} \hat{\mu}.$$

*Proof.* See the Appendix.

*Q.E.D.*

From this Proposition, the GDP growth rate is also hump-shaped in  $\mu$ , and the growth rate of GDP per capita is constant at 1.

Since the analysis is too complicated if both the time cost and the monetary cost are combined, I analyze the two cases separately. However, it is fair to say that if the monetary cost is more crucial for raising children than the time cost, then we will obtain results similar to those in this section; otherwise, we will see a pattern similar to that in Section 4.

## 7 Concluding remarks

This paper considers a three-period OLG model in which a young agent faces a borrowing constraint against human capital accumulation, and a middle-aged agent determines the fertility rate endogenously. The main purpose of this paper is to examine how the borrowing constraints affect the growth rates of the population, GDP, and GDP per capita on a BGP equilibrium. The paper's results show that the fertility rate declines as the borrowing of young agents increases, which has been a growing concern in some developed countries such as the U.S., Japan, and so on. Furthermore, the paper showed a case where the GDP growth rate decreases as the borrowing constraints are relaxed, whereas the growth rate of GDP per capita increases. Thus, relaxing the borrowing constraints may not be as detrimental as indicated by the previous literature, for example, Kitaura (2012).

Although the huge burden of student loans is becoming an important issue in terms of the future economy, to the best of my knowledge, there is little research on this matter. This paper assumes homogeneous agents, and hence, all young agents had to borrow. This is quite an extreme assumption, because not all people need to borrow to go to college or university. One possible extension of this paper is to consider agents with different wealth levels and examine how the borrowing constraints affect the economy in such cases.

## A Appendix

### A.1 Proof of Lemma 3.1

*Proof.* (i)  $\bar{\mu}$  can be rewritten as  $\frac{1}{\frac{1}{\eta} + \frac{\gamma(1-\eta)}{\eta(1+\beta)}}$ . As  $\beta$  increases, the denominator strictly decreases. Thus,  $\bar{\mu}$  is strictly increasing in  $\beta$ . Since the denominator of  $\bar{\mu}$  is increasing in  $\gamma$ ,  $\bar{\mu}$  is strictly decreasing in  $\gamma$ . (ii) Since the numerator of  $\bar{\mu}$  is strictly increasing in  $\eta$  and its denominator is strictly decreasing in  $\eta$ ,  $\bar{\mu}$  is strictly increasing in  $\eta$ . Plugging  $\eta = 0$  and  $\eta = 1$  into  $\bar{\mu}$  proves the latter result. *Q.E.D.*

## A.2 Proof of Proposition 4.1

*Proof.* When  $\mu < \hat{\mu}$ , the borrowing constraint binds. Thus, the population growth rate is  $\bar{n}^b = \frac{\gamma(1-\mu)}{\phi(1+\beta+\gamma)}$ . It is not difficult to check  $\frac{d\bar{n}^b}{d\mu} < 0$ . When  $\mu \geq \hat{\mu}$ , the borrowing constraint does not bind anymore. Hence, the population growth rate is  $\bar{n}^{nb} = \frac{\gamma(1-\eta)}{\phi[1+\beta+\gamma(1-\eta)]}$ . This does not depend on  $\mu$ . *Q.E.D.*

## A.3 Proof of Proposition 4.2

*Proof.* Since I want to know how  $g^b$  changes as  $\mu$  changes, I take the derivative of  $\frac{g^b}{Z^b}$  with respect to  $\mu$ .

$$\frac{d\left(\frac{g^b}{Z^b}\right)}{d\mu} = -Q^{\frac{\eta}{1-\alpha(1-\eta)}} + \frac{(1-\mu)\eta}{1-\alpha(1-\eta)} Q^{\frac{\eta}{1-\alpha(1-\eta)}-1} \frac{(1-\alpha)(1+\beta)\alpha\mu^{-\alpha} - \alpha\mu^{1-\alpha}[(1-\alpha)(1+\beta)+\gamma]}{\{(1+\beta)\alpha + \mu[(1-\alpha)(1+\beta)+\gamma]\}^2},$$

where  $Q := \frac{\mu^{1-\alpha}}{(1+\beta)\alpha + \mu[(1-\alpha)(1+\beta)+\gamma]}$ . The sign of this is equivalent to the sign of

$$-\frac{\mu^{1-\alpha}}{(1+\beta)\alpha + \mu[(1-\alpha)(1+\beta)+\gamma]} + \frac{(1-\mu)\eta}{1-\alpha(1-\eta)} \frac{(1-\alpha)(1+\beta)\alpha\mu^{-\alpha} - \alpha\mu^{1-\alpha}[(1-\alpha)(1+\beta)+\gamma]}{\{(1+\beta)\alpha + \mu[(1-\alpha)(1+\beta)+\gamma]\}^2}.$$

Moreover, the sign of this is equivalent to the sign of

$$m(\mu) := -(1-\alpha)[(1-\alpha)(1+\beta)+\gamma]\mu^2 - \alpha\{[1-\alpha(1-\eta)](1+\beta) + \eta(1-\alpha)(1+\beta) + \eta[(1-\alpha)(1+\beta)+\gamma]\}\mu + \eta(1-\alpha)(1+\beta)\alpha.$$

The function  $m$  is hump-shaped, satisfies  $m(0) > 0$ , and takes the maximum at some  $\tilde{m} < 0$ . Note that  $m(1) = -(1-\alpha)[(1-\alpha)(1+\beta)+\gamma] - \alpha\{[1-\alpha(1-\eta)](1+\beta) + \eta(1-\alpha)(1+\beta) + \eta\gamma\} < 0$ . Thus, there exists a unique  $\tilde{\mu} \in (0, 1)$  such that  $\frac{dg^b}{d\mu} > (<)0$  for  $\mu < (>)\tilde{\mu}$ . *Q.E.D.*

## A.4 Proof of Proposition 4.3

*Proof.* To know the sign of  $\frac{d\hat{g}^b}{d\mu}$ , I focus on how  $\frac{\mu^{1-\alpha}}{(1+\beta)\alpha + \mu[(1+\beta)(1-\alpha)+\gamma]}$  changes with  $\mu$ . Taking the derivative of  $\frac{\mu^{1-\alpha}}{(1+\beta)\alpha + \mu[(1+\beta)(1-\alpha)+\gamma]}$  with respect to  $\mu$ , I obtain

$$\begin{aligned} & \frac{(1-\alpha)\mu^{-\alpha}}{(1+\beta)\alpha + \mu[(1+\beta)(1-\alpha)+\gamma]} - \frac{\mu^{1-\alpha}[(1+\beta)(1-\alpha)+\gamma]}{\{(1+\beta)\alpha + \mu[(1+\beta)(1-\alpha)+\gamma]\}^2} \\ &= \frac{\mu^{-\alpha}\alpha}{\{(1+\beta)\alpha + \mu[(1+\beta)(1-\alpha)+\gamma]\}^2} [(1-\alpha)(1+\beta) - \{(1-\alpha)(1+\beta)+\gamma\}\mu]. \end{aligned}$$

Hence, if  $\mu < \hat{\mu} = \frac{(1-\alpha)(1+\beta)}{(1-\alpha)(1+\beta)+\gamma}$ ,  $\frac{d\hat{g}^b}{d\mu} > 0$ . Q.E.D.

### A.5 Proof of Lemma 4.1

*Proof.* Plugging  $\hat{\mu}$  into  $m(\mu)$ , I have

$$m(\hat{\mu}) = -(1-\alpha)[(1-\alpha)(1+\beta) + \gamma]\hat{\mu}^2 - \alpha\{[1-\alpha(1-\eta)](1+\beta) + \eta(1-\alpha)(1+\beta)\}\hat{\mu} < 0.$$

Since  $m(\tilde{\mu}) = 0$  and  $m(\mu) < 0$  for any  $\mu > \tilde{\mu}$ ,  $\tilde{\mu} < \hat{\mu}$ . Q.E.D.

### A.6 Proof of Proposition 6.1

*Proof.* Taking the derivative of  $\bar{n}^b$  with respect to  $\mu$ , I have

$$\frac{d\bar{n}^b}{d\mu} = \frac{M\mu^{-\alpha}}{[\alpha + \mu\theta(1-\alpha)]^2} Q(\mu),$$

where  $Q(\mu) := -\theta(1-\alpha)^2\mu^2 - \alpha[1 + (1-\alpha)(1+\theta)]\mu + \alpha(1-\alpha)$ . Note that  $Q(0) = \alpha(1-\alpha) > 0$  and  $Q(1) = -\theta(1-\alpha) - \alpha(1-\alpha)(1+\theta) - \alpha^2 < 0$ . In addition,  $Q(\mu)$  reaches its maximum at some negative  $\mu$ . Thus, there exists a unique  $\hat{\mu} \in (0, 1)$  such that  $Q(\mu) > 0$  for  $\mu < \hat{\mu}$ ,  $Q(\hat{\mu}) = 0$ , and  $Q(\mu) < 0$  for  $\mu > \hat{\mu}$ . This completes the proof. Q.E.D.

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